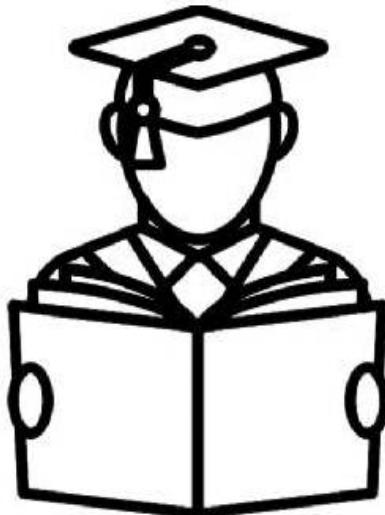


चौधरी PHOTOSTAT

"I don't love studying. I hate studying. I like learning. Learning is beautiful."



"An investment in knowledge pays the best interest."

Hi, My Name is

Civil Engineering
for GATE/IES
(MADE EASY)

STRUCTURE ANALYSIS

S.A

I.E.S |

GATE

8 to 10 marks

↓
Objective: Conventional

15 to 20

40 marks

ques

- Stability & Degree of indeterminacy - Ob
- Influence line diagrams - Ob + Conv.
- Roches - Ob
- Method of analysis - Strain Energy -
 - ↳ Slope deflection
 - ↳ moment distribution
 - ↳ MatrixOb + Conv.
- Analysis of trusses - Determinate
 - ↳ Indeterminate
 - ↳ Deflection of beam jointOb + Conv.

Reference book -

(1) R Theory of Structure - S. Ramamurtham

[Strain Energy, Moment Distribution
Truss Analysis]

2) Structure Analysis - Negi & Jangid.

[Slope deflection, Influence line]

(3) Theory of Structure - Gupta & Panekar.

[Basic concept, Stability & Determinate
Matrix method]

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Stability And Incliminity.

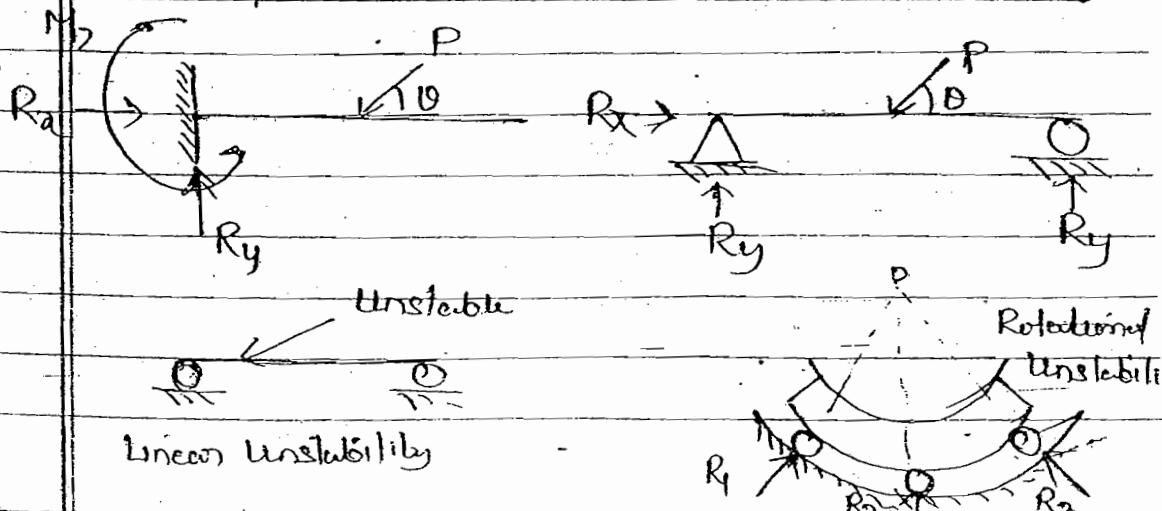
Stability

External Internal
 [Related to Support Cond'n] [Related to Geometry]

External Stability -

Large displacement of support or entire structure are not permitted. Therefore there should be enough fix'n at support to prevent movement and also fix'n should be arranged in appropriate way so that there will not rigid body motion however elastic deflection in members may occur.

In plain structure (2D) there should be a min. of 3 independent external fix'n which should be non parallel and non concurrent.



For Stability of 2D structure

Following 3 condⁿ of static equilibrium Should be satisfied

Co
ptur

- (1) $\sum F_x = 0$ To prevent Δ_x
- (2) $\sum F_y = 0$ To prevent Δ_y
- (3) $\sum M_z = 0$ To prevent Θ_z

In case of 3D structure, there should be min. of 16 independent external force to prevent rigid body displacement at support.

The displacement to be prevent are Δ_x , Δ_y , Δ_z , Θ_x , Θ_y & Θ_z .
Therefore there will be 6 (Six) equations of static equilibrium.

- (1) $\sum F_x = 0$ To prevent Δ_x
- (2) $\sum F_y = 0$ " Δ_y
- (3) $\sum F_z = 0$ " Δ_z
- (4) $\sum M_x = 0$ " Θ_x
- (5) $\sum M_y = 0$ " Θ_y
- (6) $\sum M_z = 0$ " Θ_z

To d 2D - plane structure

3D - Space structure

In 3D structure for stability

all the mem. should non plane parallel and non concurrent.

Internal Stability -

No part of the structure can move relative to the other part so that geometry of the structure is preserved. However small elastic deformation are permitted. To preserve the geometry enough no. of members and their unique arrangement is required. For geometric stability there should not be formed of condⁿ of mechanism (there should not be three collinear things).

For 2D truss the min. no. members needed for geometric stability is

$$M = 2J - 3$$

and for 3D truss -

$$M = 3J - 6$$

The all the members should be arranged such that truss is deviated in triangular shape. There should not be rectangular or poly gonat block.



$$J = 6$$

eg.
7

No. members needed for geometric stability.

$$m = 2J - 3$$

$$= 2 \times 6 - 3$$

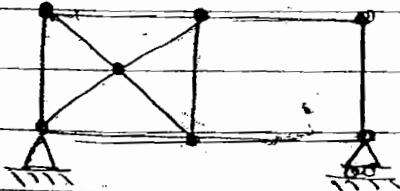
$$= 9$$

No. members p.f. 1

$$= 7$$

eg.
8

Hence, above truss is geometrically unstable because such is called "Deficient Structure".

eg.
8

$$m = 9$$

$$= 2J - 3$$

In above Case arrangement of members is not adequate hence right panel is unstable and left panel is over stable.

eg.
9

For geometric stability all panels of truss or members should be stable.

HYDROLOGY IRRIGATION

Hydrology :-

Hydrology is the science which deals with the occurrence, circulation & distribution of water on Earth Surface and its atmosphere.

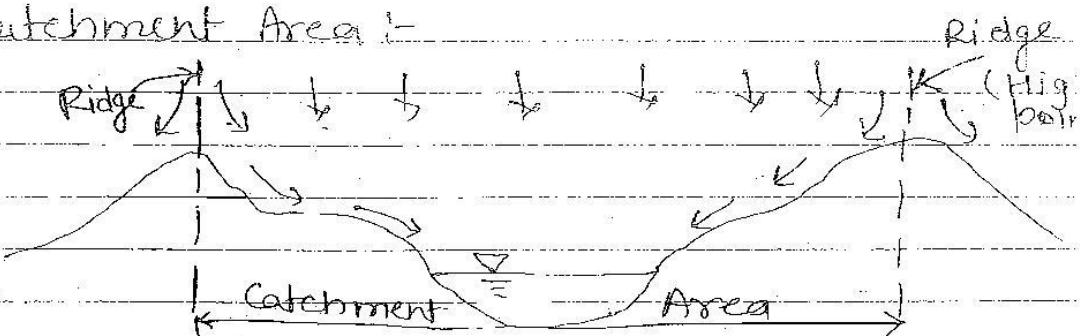
Hydrological Cycle:-

This is a cycle in which water moves from one phase to another phase.

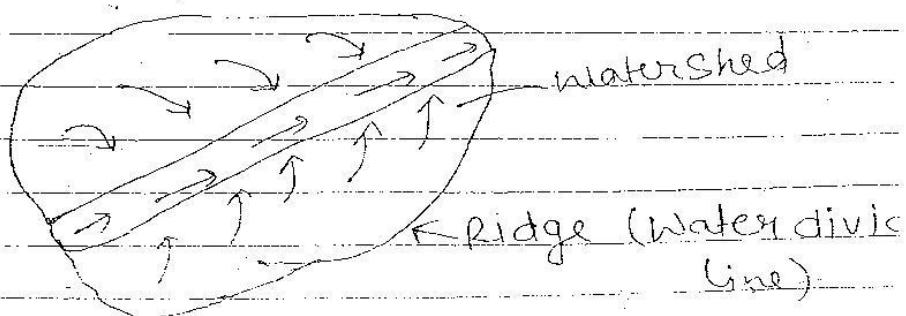
Residence time :-

This is the time taken by a water particle in crossing one particular face of hydrological cycle.

Catchment Area :-



The area draining into a river or stream is called the catchment area for that particular stream or river.



Hydrology :-

Hydrology is the science of water which deals with the occurrence, circulation & distribution of water on Earth Surface and its atmosphere.

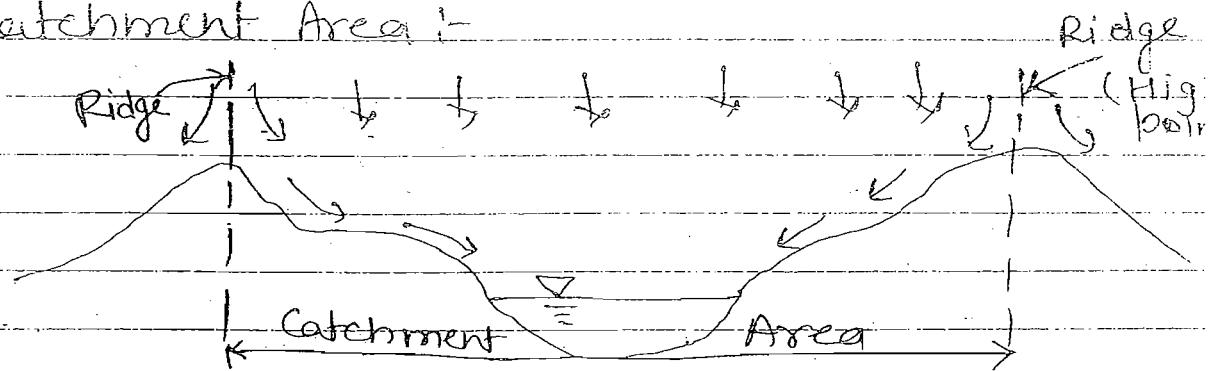
Hydrological Cycle:-

This is a cycle in which water moves from one phase to another phase.

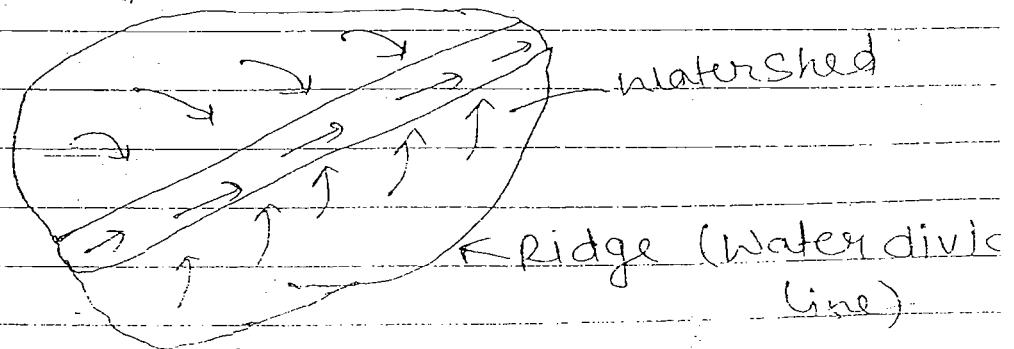
Residence time :-

This is the time taken by a water particle in ~~pass~~ crossing one particular phase of hydrological cycle.

Catchment Area :-



The area draining into a river or stream is called the catchment area for that particular stream or river.



Catchment area is also called as watershed.

Ridge: It is the line which demarcates one catchment area from its neighbouring area. This is also called as water divide line or divide line!

Water budget Equation:

This Eqⁿ is based on the law of conservation of mass and according to it,

$$\text{Mass Inflow} - \text{Mass outflow} = \text{Change in Storage}$$

A lake has a water surface elevation of 103.2 m above datum. In a month the lake receives an avg. inflow of 6 cumecs and in the same period outflow from the lake was 6.5 cumecs. In that month the lake receives the rainfall of 145 mm & evaporation from lake surface is estimated as 6.1 cm. Calculate the water surface of elevation at the end of month. Surface area of lake is 5000 hectare.

So) Inflow = 6 cumecs + 145 mm

$$\text{Outflow} = 6.5 \text{ m cumecs} + 6.1 \text{ cm}$$

$$\text{Inflow} = \left(6 \frac{\text{m}^3}{\text{s}} \times 60 \times 60 \times 24 \times 30 \right) / \text{Area}$$

$$+ .145 \text{ m} =$$

one year is called Annual Rainfall.

Avg. annual rainfall is the avg. value of annual rainfall for the last 35 years.

Index of Wetness :-

This Index is used to find the rainfall variation for the particular year and it is calculated as.

$$\text{Index of Wetness} = \frac{\text{Rainfall in a year}}{\text{Avg. Annual Rainfall}} \times 100$$

$$\text{eg. Index of Wetness} = \frac{90\text{cm} \times 100}{120\text{cm}} = 75\%$$

This shows that the rainfall deficiency is 25%.

If rainfall deficiency is in b/w 30-45% It is called large deficiency.

If it is in b/w 45-60% It is called serious deficiency.

If it is more than 60% It is disaster deficiency.

If Index of Wetness is equal to 100% It indicates that rainfall is normal.

If this Index is less than 100% It is called a bad year.

If this Index is more than 100% Then It is called a good year.

Drought :-

This is the climatic situation which is characterised by

Drought can be classified as follows:-

1 Meteorological drought:-

This represents deficiency in precipitation. If the decrease in precipitation is more than 25%. It is called

If it is in b/w 25% - 50% it is called moderate drought

If it is more than 50% it is called

An year is called drought year if area affected by drought is more than 25% of the total area of the country.

If drought occurs in an area with a probability of .2 to .4 then that area is called drought prone area. and If this probability is greater than .4 it is called chronologically drought prone area.

2 Hydrological drought:-

This refers to the below avg. value of stream flow, water content in lakes, reservoir, underground water

3 Agricultural drought:-

This type of drought is characterised by deficiency of moisture available for a plant growth. This can be calculated using a

$$\begin{aligned}
 \text{outflow} &= 6.5 \frac{\text{m}^3}{\text{s}} \times (60 \times 60 \times 24 \times 30) / 5000 \\
 &\quad + 0.061 \text{ m} \\
 &= \left(\frac{6 \times 60 \times 60 \times 24 \times 30}{5000} + 0.145 \right) - \left(\frac{6.5 \times 60 \times 60 \times 24 \times 30}{5000} \right. \\
 &\quad \left. + 0.061 \right) \\
 &= 0.456 - 0.398 \\
 103.2 + 0.456 - 0.398 &= 103.258 \text{ m}
 \end{aligned}$$

Precipitation :-

Precipitation denotes all forms of water or moisture that reaches the Earth Surface.

following are the different forms of precipitation:-

1) Rain :-

This is the principal mode of precipitation in India. This denotes water droplets & size ranging from 0.5 mm to 6mm. On the basis of Intensity rainfall is classified as

$0 - 2.5 \text{ mm/hr}$ → Light rain

$2.5 - 7.5 \text{ mm/hr}$ → Moderate

$> 7.5 \text{ mm/hr}$ → Heavy Rain

NOTE : In India rainfall data is collected every day at 8:30 A.M and if this rainfall is more than 2.5 mm on a particular day then that day is called rainy day.

from mm → we can calculate total vol. by multiplying the area \times depth.

and its discharge can be calculated m^3/s

SURVEYING

Surveying :-

Introduction:-

Earth is oblate spheroid.

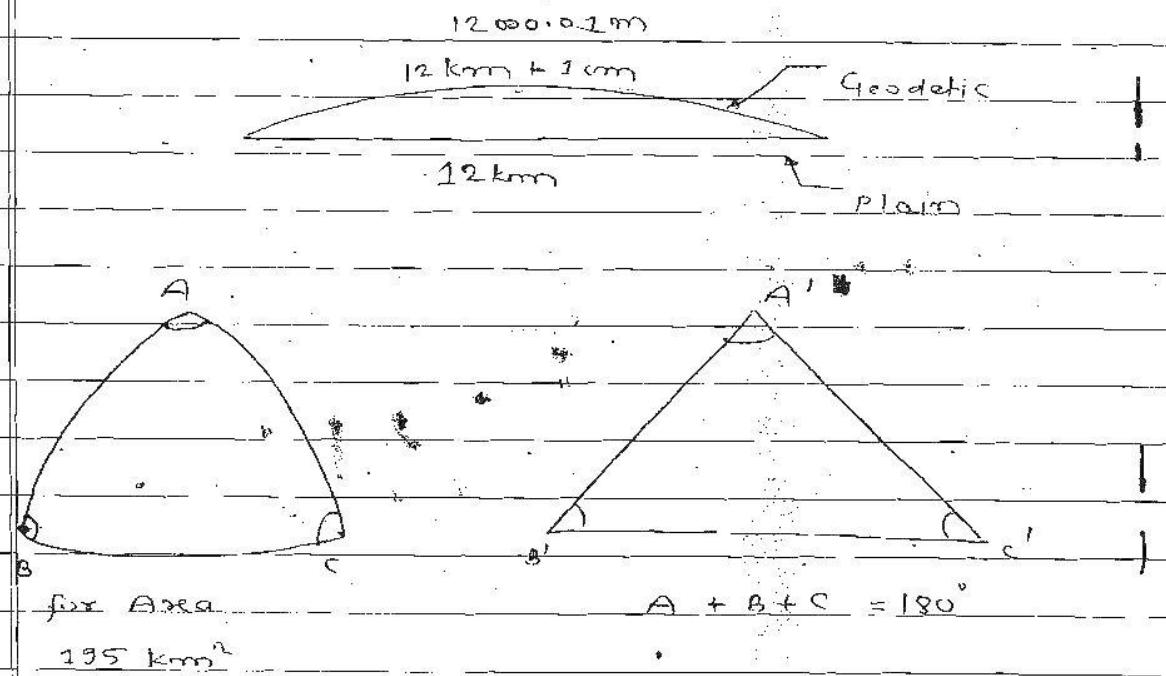
Polar Axis = 12713.80 km (less)

Equatorial axis = 12756.75 km (more)

Difference = 42.75 km (0.34%)

1. Plane Surveying :- If earth curvature is not considered (for small area)

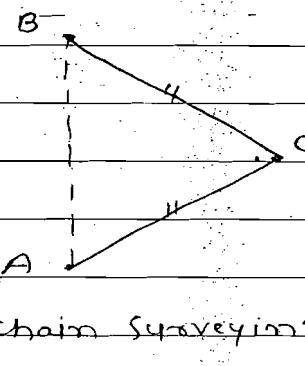
2. Geodetic Surveying :- Earth curvature is considered (suitable for large area)



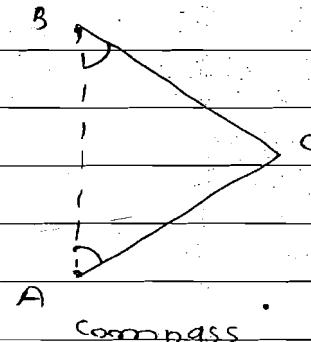
* Principle of Surveying

In this method major control points are fixed and measured with higher accuracy minor details can be taken even with less precision. In this case mistakes/error in taking minor details will not reflect in major measurement.

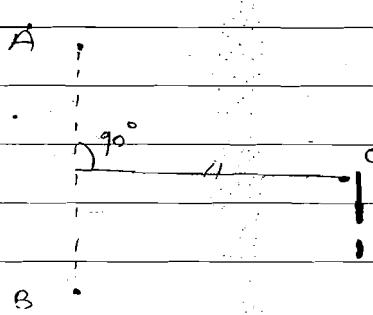
2) Location of a point:- w.r.t. two reference point.



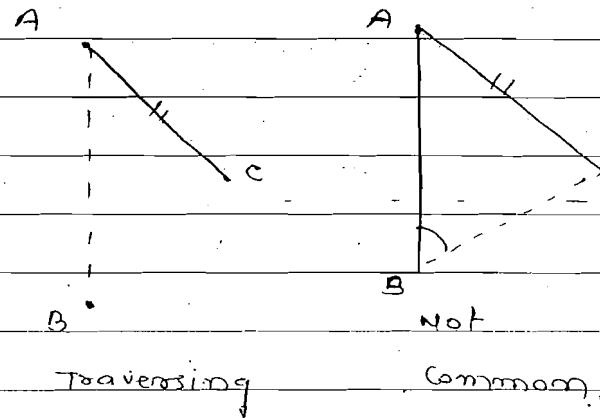
chain surveying



compass



offset method



traversing

not

common

Accuracy and Errors :-

1. Precision:- Degree of perfection used in a measurement.
Using correct instrument, correct manner of measurement.
2. Accuracy:- Degree of perfection obtained in a measurement
(The measured value should be near to the true value)
3. True value:- The actual value of a quantity.

True Error - Difference between a measured value and true value.

4. Discrepancy :- Difference between the two measurements of a same quantity.

Sources:- [of errors]

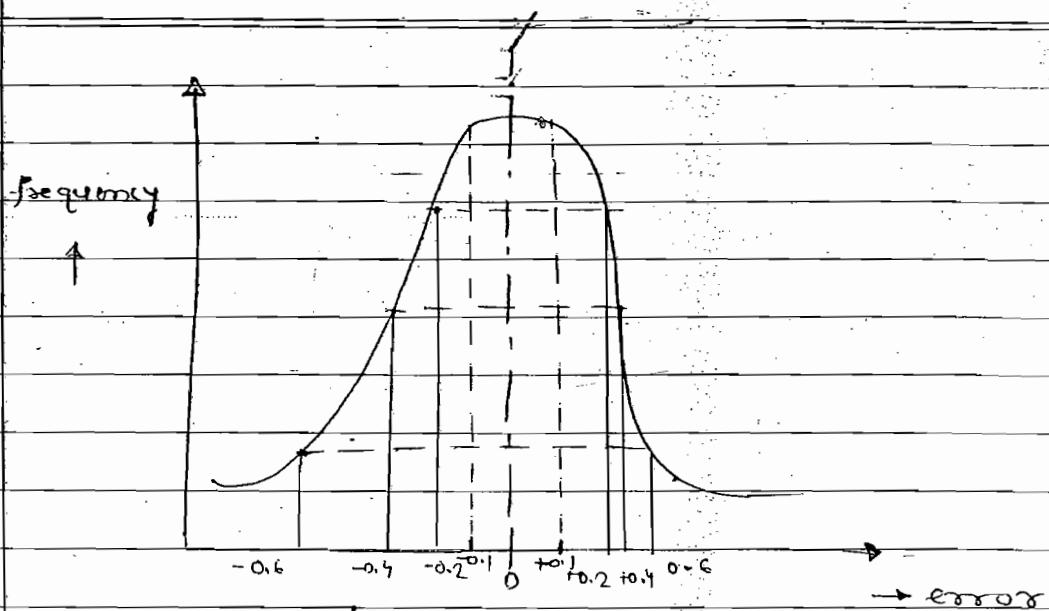
1. Instrumental - Due to faulty instrument
2. Personal - Error in reading or writing
3. Natural - Temperature, wind, humidity etc.

Type of Errors:-

1. Mistakes :- Human errors. (may be + or - ve)
2. Systematic Errors :- (Cumulative errors):
That always occurs in same direction.
(Always +ve or Always -ve)
3. Accidental Errors [compensating error or Random Error]
May occurs some times in one direction
(say +ve) and sometimes in others (say -ve)

Accidental errors follow a rule -

"Law of probability"

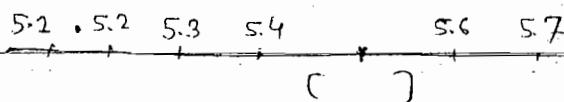


As per this law of errors:-

1. Small errors are more frequent than large errors (Frequency of large errors shall be less)
2. The +ve and -ve value errors of same value has equal frequency.

Most Probable Value (MPV) :-

The value of measurement which has more chance of being the correct value of the quantity is called Most Probable Value.)



Principle of least square:-

Most probable value of a quantity is that value for which sum of square of errors is least.

case-1 : if weight of each measurement is one-

$x_1, x_2, x_3, \dots, x_m$ are different measurements

of same quantity,

if x is most probable value =

	Errors	Square of Errors
x_1	$(x - x_1)$	$(x - x_1)^2$
x_2	$(x - x_2)$	$(x - x_2)^2$
x_3	$(x - x_3)$	$(x - x_3)^2$
x_4	$(x - x_4)$	$(x - x_4)^2$
x_m	$(x - x_m)$	$(x - x_m)^2$

As per principle of least square :-

$$y = (x - x_1)^2 + (x - x_2)^2 + \dots + (x - x_m)^2$$

for y to be least value -

$$\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 2(x - x_1) + 2(x - x_2) + \dots + 2(x - x_m) = 0$$

$$2 [x_m - (x_1 + x_2 + x_3 + \dots + x_m)] = 0$$

$$x = \frac{x_1 + x_2 + x_3 + \dots + x_m}{n} \quad \text{Most probable value.}$$

MPV = Mean Value

Case-II: When different measurement have weights

Measurement	Weight	MPV	Error	Square of Error	Total square error
x_1	w_1		$(x - x_1)$	$(x - x_1)^2$	$w_1(x - x_1)^2$
x_2	w_2		$(x - x_2)$	$(x - x_2)^2$	$w_2(x - x_2)^2$
-	-	x	-	-	-

As per principle of least square -

$$y = w_1(x-x_1)^2 + w_2(x-x_2)^2 + \dots = \text{least}$$

$$\frac{dy}{dx} = 2w_1(x-x_1) + 2w_2(x-x_2) + \dots = 0$$

$$\therefore x = (w_1x_1 + w_2x_2 + \dots + w_m x_m) \\ (w_1 + w_2 + w_3 + \dots + w_m)$$

MPV = weighted Average.

1. The probable error of single measurement :-

$$E_s = \pm 0.6745 \sqrt{\frac{\sum v^2}{n-1}}$$

where - v = difference between any single observations and mean of the series
 n = number of observations.

2. The probable error of mean of a number of observations :-

$$E_m = \pm 0.6745 \sqrt{\frac{\sum v^2}{n(n-1)}} = E_s \sqrt{\frac{1}{n}}$$

3. If measurements of have different weights
 $(x_1-w_1, x_2-w_2, \dots)$

- (a) Probable error of single measurement

$$E_s = \pm 0.6745 \sqrt{\frac{\sum (wv^2)}{n-1}}$$

- (b) Probable error of any observation of wt w_i .

W.M. :-

I. Cement concrete :-

Cement + Sand + Coarse agg
(fine agg)

a) Young's modulus of elasticity (E_c) as per IS 456 : 2000

$$E_c = 5000 \sqrt{f_{ck}}$$

where f_{ck} = characteristic strength.

b) Characteristic strength f_{ck} :

The value of strength below which not more than 5% of test results are expected to fall.

If 100 cubes are tested

→ 21.0, 19.0, 21.5, ...

Arrange all values in increasing order

16.5, 17, 18, 18.3, 19, 20, 21, 21, 21, 21

less 5% f_{ck} more 5%

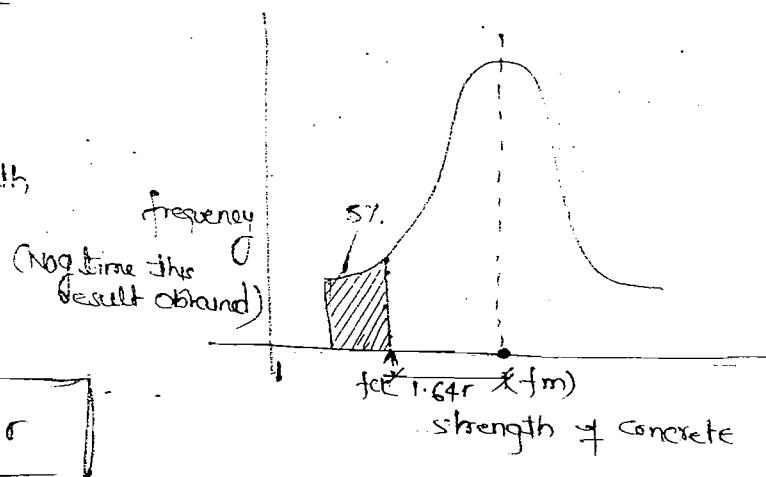
Probabilistic curve :-

f_{ck} = characteristic strength

f_m = mean strength

$$f_m > f_{ck}$$

$$f_m = f_{ck} + 1.64\sigma$$



σ - > 3 to 4 (standard value)

$$\Rightarrow 4 \text{ N/mm}^2 = M_{20} \Sigma M_{2j} - 5 \text{ N/mm}^2 = M_{30} \Sigma M_{50}$$

If $\leq 5\%$ not more than 50% of test results are expected to fall below characteristic strength, in this case $f_{ck} = f_m$

Confidence limit :-

maxim probability of a particular test result shall be within a range of

$(f_m - 1.64\sigma)$ to $f_m + 1.64\sigma$ → these two limits are called confidence limits.

The probability γ of lying the test result within confidence limit is called confidence level.

Grade of concrete :-

As per IS-456.

- Ordinary concrete — M10 to M20
- Standard Concrete — M25 to M55
- High Strength Concrete. — M60 to M80

M = mix

25 = characteristic strength.

→ For RCC min grade recommended = M20

i) Young's modulus → effects of creep diff type :-

ii) Young's modulus of elasticity at origin :-

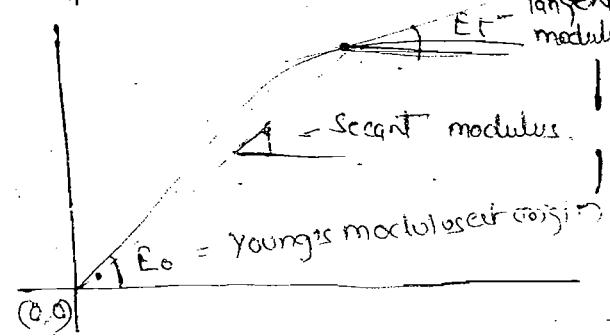
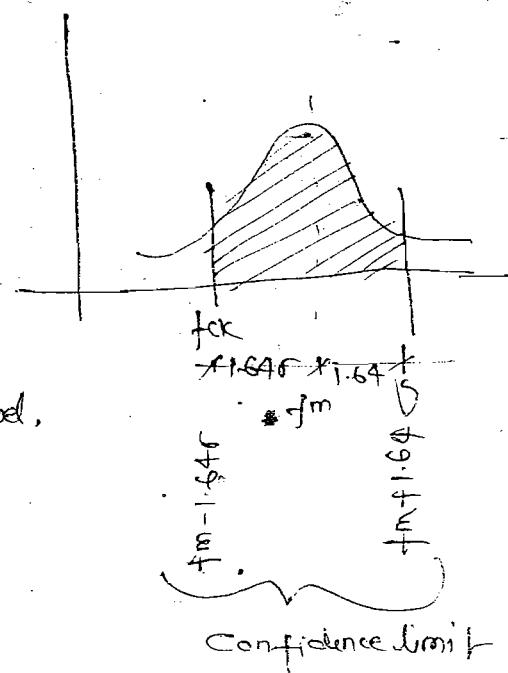
Slope of stress strain curve at origin

$$E_0 = E_c = 5000 \sqrt{f_{ck}}$$

Secant modulus of elasticity
slope of line joining any point (with origin)

iii) Tangent modulus of elasticity

Slope of tangent at any point of the curve.



Effect of Creep: Long term modulus of elasticity

$$E_{cl} = \frac{E_c}{1+\theta} = \frac{5000\sqrt{f_{ck}}}{1+\theta}$$

Where θ = creep co-efficient = $\frac{\text{Creep strain}}{\text{elastic strain}}$

As per IS: 456: (Pr. 6.25.1 / page-16)

Age of loading	Creep Loading
7 day	0.2
28 days	1.6
1 year	1.1

E_{cl} reduces due to effect of creep

Tensile strength of concrete in bending =

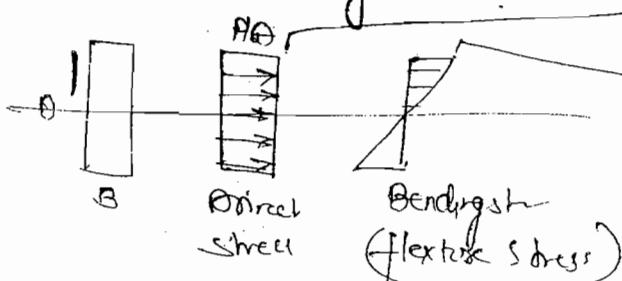
$$f_{cr} = 0.7\sqrt{f_{ck}}$$

Grade	M20	M25	M30	M35	M40
f_{cr}	3.13	3.5 N/mm ²	3.83	4.14	4.43

B. Permissible stress in concrete (for NCM)

Grade	Tensile strength		Compressive strength		f_{cbc}
	Direct	Bending	Direct	Bending	
M15	0.0	2.71	4.0	5.0	
M20	0.8	3.13	5.0	7.0	
M25	3.2	3.5	6.0	8.5	
M30	3.6		8.0	10.0	
M35	4.0		9.0	11.5	
M40	4.4		10.0	13.0	
N/mm ²					

Direct & bending stress



* Chance of failure is more in direct stress as section is subjected to some shear.

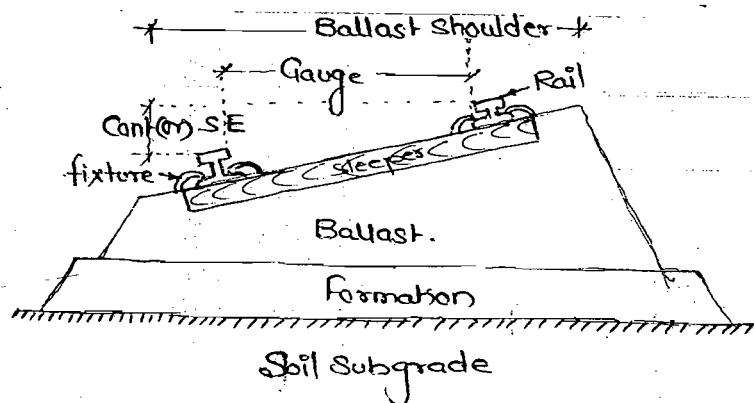
* Chance of failure is less, as only top & bottom fibres at maximum stress. Stress in other fibres are less than failure value.

Introduction :-

- Cross-section of a railway track (on a curved track)

2. Gauge :-

- BG [Broad Gauge] - 1.676m
- MG [Meter Gauge] - 1.0m
- NG [Narrow Gauge] - 0.762m
- LG [Feedertrack Gauge] - 0.61m
(or)
Light gauge

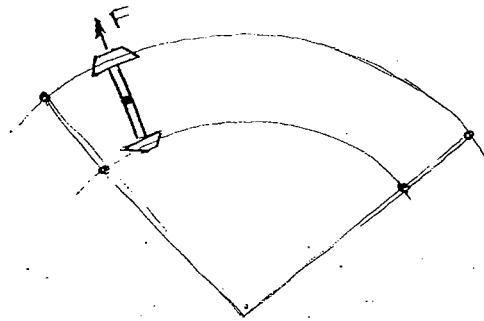
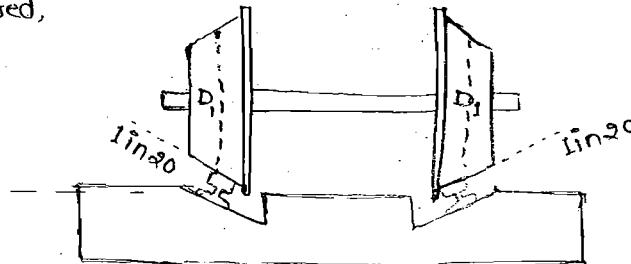


3. Canning of wheels :-

Canning of wheels are made tapered, with slope of 1 in 20, so that diameter of wheel is different at different cross section.

Purpose :-

- To keep the train just in central position during movement
- To adjust the distance travelled on two rails on a curved track where longer distance is required to be travelled on other track.
- To reduce the wear & tear of rail & wheels.



Theory :- When on a straight track, when train moves sideways in any direction, diameter of wheel over one rail will increase, & it will decrease over another thus automatically the train is diverted back in its original central position due to longer distance travelled over one rail as compared to another.

4. Welded Rail :-

LWR - Long welded rail :- Rails are welded to avoid expansion joint. In case of long welded rails, elongations of rails are not allowed, prevented by fixtures over sleepers. To overcome the stresses developed in rails a minimum length of LWR is required.

If minimum length of LWR = L .

Due to $T^{\circ}\text{C}$ temp. increase, Increase in length

$$\delta L = LD\alpha T$$

If δL elongation is not allowed strain developed

$$e = \frac{\delta L}{L} = \frac{LD\alpha T}{L} = D\alpha T$$

Stress developed

$$\frac{\text{stress}}{\text{strain}} = E$$

$$\text{stress} = E \times \text{strain}$$

$$= E D\alpha T$$

If A_s = cross section of Rail force developed in rail section $F = \text{stress} \times A_s$

$$F = A_s E D\alpha T \quad \text{--- (1)}$$

If R = resistance offered by one sleeper

Min^m no of sleepers =

Minimum no. of sleepers required to prevent

F force

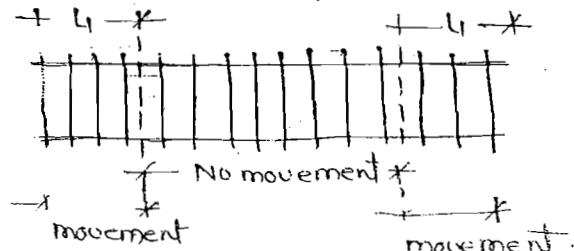
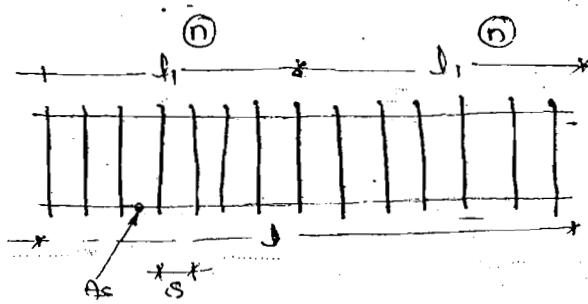
$$n = \frac{F}{R} = \frac{A_s \cdot E D\alpha T}{R}$$

length of rail required in one direction

$$J_1 = (n-1) S$$

Total minimum length of LWR so that central portion does not move

$$L = 2J_1$$



1) Determine the minimum theoretical length of LWR, beyond which central portion of a 56kg rail would not be subjected to longitudinal movement, due to 30°C temp variation use :-

1. Rail - Cross area 66.15cm^2 ; $E_s = 2.1 \times 10^6 \text{kg/cm}^2$; $\alpha = 11.5 \times 10^{-6}/^{\circ}\text{C}$

2. Sleepers - sleeper spacing = 60cm

Average resistance force/sleeper/rail = 300kg

Sol:-

Due to 30°C temp increase, temp.

$$\Delta L = \alpha L \Delta T$$

If above elongation is not allowed

$$\text{strain} = \frac{\Delta L}{L} = \alpha T$$

Stress developed due to above strain

$$P_e = E \epsilon T$$

$$= 2.1 \times 10^6 \times 11.5 \times 10^{-6} \times 30$$

$$= 724.5 \text{ kg/cm}^2$$

Force developed in rail section = $A_s \times P_e$

$$= 66.15 \times 724.5$$

$$= 47925.675 \text{ kg}$$

minimum no of sleepers required to prevent this above force

$$n = \frac{\text{force}}{\text{resistance}} = \frac{47925.675}{300}$$

$$= 159.75 \approx 160 \text{ nos.}$$

minimum length required in one direction

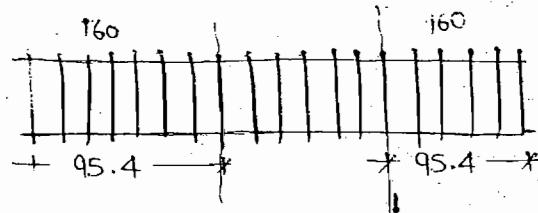
$$= (n-1) \cdot s$$

$$= (160-1) \times 0.6 = 95.4 \text{ m.}$$

minimum length of LWR = $2L_1$

$$= 2 \times 95.4$$

$$= 190.8 \text{ m.}$$



Composite Sleeper Index

* For wooden sleepers \therefore CSI is an index to determine the stability of a particular timber to be used as sleeper.

$$CSI = \frac{S+10H}{20} \quad \text{where } S = \frac{\text{Strength index of timber at } 12\% \text{ moisture content}}{} \quad H = \text{Hardness index of timber.}$$

Minimum value	CSI
Track sleeper	783
Crossing	1359
Bridge	1455

13-14m

* Sleeper density \therefore

No. of sleepers used for one rail length denoted by $(n+s)$

Varies from $(n+3)$ to $(n+6)$

Ex:- If sleeper density for a broad gauge is $(n+5)$, how many sleepers shall be required for 1 Km track.

Sol:- $= 19.8 \text{ m} \approx$

length of one rail = 13 m

Sleeper density $= (n+5) = (13+5) = 18 \text{ Nos}$

for 1 Km length

$$= \frac{1000}{19.8} \times 18 = 1406 \text{ Nos.}$$

Geometrical Design

1. Speed of Train \therefore

Maxm speed allowed on a track is decided considering following :-

- Maxm speed sanctioned by railway board.
- On curve speed calculated by martin's formula [Safe speed on curve]
- Maxm speed as per Cant formula
- Maxm speed as length of transition curve.

Safe Speed on curve [By Martin's formula]

1. On Transitioned curved

a) On BG / MG Track

$$V_{max} = 4.35 \sqrt{R-67}$$

b) For NG Track

$$V_{max} = 3.65 \sqrt{R-6}$$

2. on non-transitioned curved

Speed = 80% of for transitioned curve.

3. For high speed train

$$V_{max} = 4.58 \sqrt{R}$$

Radius of curve / Degree of curve :-

Angle made at the centre of curve by
1 chain length is called degree of
curve [D°].

1. For 30m chain length.

$$\frac{30}{2\pi R} = \frac{D^\circ}{360}$$

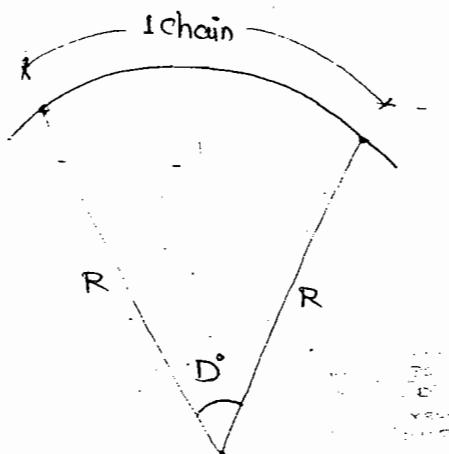
$$D^\circ = \frac{30 \times 360^\circ}{2\pi R} = \frac{1718.9}{R} \approx \frac{1720}{R}$$

2. For 20m chain length

$$\frac{20}{2\pi R} = \frac{D^\circ}{360}$$

$$D^\circ = \frac{20 \times 360^\circ}{2\pi R} = \frac{1145.9}{R} \approx \frac{1150}{R}$$

Degree of curve	1	2	3	4	5
30m	1720m	860m	573m	430m	344m
20m	1150m	575m	383m	288m	230m



Max^m limit of degree of curve :-

BG	- 10°
MG	- 16°
NG	- 40°

3. Versine of Curve :-

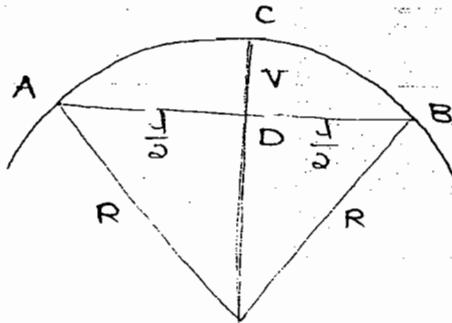
$$AD \times DB = CD \times DE$$

$$\frac{J}{2} \times \frac{J}{2} = V \cdot (2R - V)$$

$$= V \cdot 2R$$

$$\therefore V = \frac{J^2}{8R}$$

$$\text{here } 2R - V \approx 2R$$

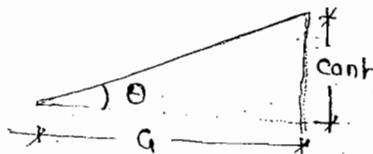
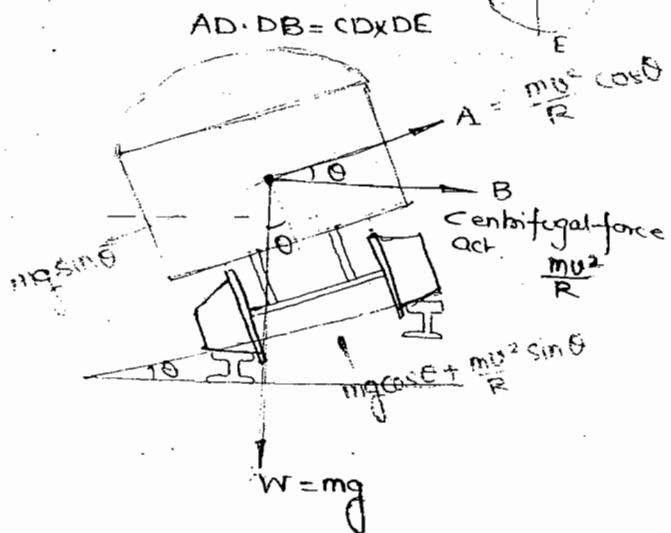


Super Elevation or Cant :-

Equating all forces in the direction of track

$$mg \sin \theta = \frac{mv^2}{R} \cos \theta$$

$$\tan \theta = \frac{v^2}{gR} \quad \text{- Slope}$$



$$\frac{\text{Cant}}{q} = \tan \theta$$

$$\text{Cant} = G \cdot \tan \theta$$

$$= G \cdot \frac{v^2}{gR}$$

$$= G \cdot \frac{(0.278V)^2}{9.81R}$$

$$\boxed{\text{Cant} = e = \frac{G \cdot V^2}{127R}}$$